Strategic Pricing in network economies: learning and algorithms

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Outline

Motivation

- Renewable energy proliferation such as solar and wind, and more carbon emission constraints implies
- New paradigms for Electricity Markets: modeling and algorithms.

Motivation

- A. Baillo et al. (2004) : New idea based on fixing scenarios for some producers instead of Nash equilibrium.
- M. Fampa et al. (2008) : Bilevel programming formulation for the problem of strategic bidding under uncertainty in a wholesale energy market.
- In D. Aussel et al. (2017) Quadratic functions and a game where the demand is known by the players.
- We extended those ideas for the case of convex piecewise linear bids, stochastic demands and learning process.

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The Network

- Graph $(\mathbf{V}, \mathbf{E})$.
- $G \subseteq \mathbf{V}$ subset of nodes associated with energy producers or firms (agents). Usually a small number. Now increasing because of small generators (renewal)
- agent ISO (Independent system operator)
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  3. After observing the vector of bids c and demands d, the central agent (ISO) runs a dispatch program subject to a number of network constraints. The results are the quantities to be produced by each generator.
  4. Firms produce as mandated by the minimum cost program and are paid at marginal cost of electricity at their nodes or a function of that.

The Dispatch Program

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$$\sum_{n \in G} c_n(q_n)$$

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$$\sum_{e \in K_n} \frac{r_e}{2} f_e^2 + d_n \leq q_n + \sum_{e \in K_n} f_e sgn(e,n), \quad n \in G \tag{1}$$

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$$\sum_{e \in K_n} \frac{r_e}{2} f_e^2 + d_n \leq \sum_{e \in K_n} f_e sgn(e,n), \quad n \notin G \tag{2}$$

The Dispatch Program

- Generation constraints: for renewal stochastic

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- Transmission Constraints:

$$0 \leq f_e \leq \bar{f}_e \tag{4}$$

The Dispatch Program

Given a demand vector $d = (d_n)_{n=1}^{N}$, the central agent (ISO) solves :

$$min \left\{ \sum_{n \in G} c_n(q_n) : (f, q) \in \Omega(d) \right\} \tag{5}$$

Where:

$$\Omega(d) = \left\{ (f, q) \in \mathbb{R}^E \times \mathbb{R}^G : (f, q) \text{ satisfies } (1) - (4) \right\}$$

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- $\Omega(d)$ is a compact convex set.
- Nodal prices are set as shadow values associated to the nodal power balances.

Outline

1. Motivation
2. The model
   - The Network and Dispatch Program
   - The Bidders
   - Convex Piecewise Linear Bids and Costs
3. Scenarios Approach
4. Conclusions and Future work
5. References

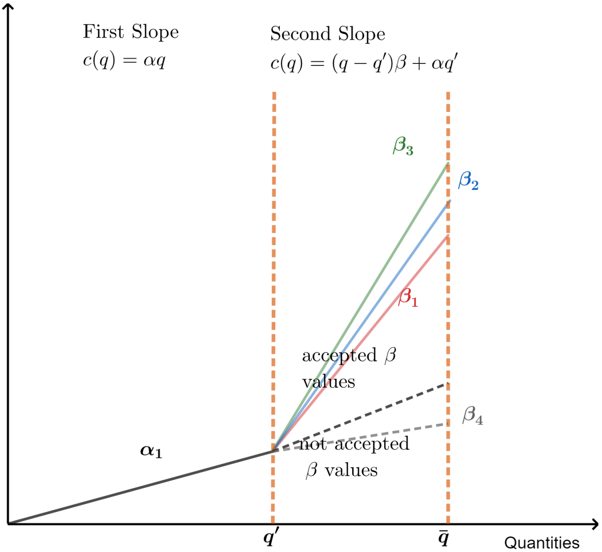
The Bidders

- Each producer bids a function $c_n(q)$.
- The payoff function is given by : $u_n(p,q) = pq - \hat{c}_n(q)$. Where $\hat{c}_n$ is the real cost function.
- Thus, the revenue is given by: $\mathbb{E}[u_n(\lambda_n(c,\cdot), q_n(c,\cdot))]$, that is,

$$\mathbb{E}[(\lambda_n(c,\cdot)q_n(c,\cdot)) - \hat{c}_n(q_n(c,\cdot))]$$

Outline

Convex Piecewise Linear Bids and Costs



Dispatch Program Algorithm

- $|G| = n$.
- Each generator k has a convex piecewise linear function defined by the slopes α^k, β^k and his break point q'_k.
- For $k = 1, \ldots, n$.
    - Denote $\alpha^{n+k} := \beta^k$.
    - Generator i can be thought as 2 generators: one with maximum capacity q'_i and linear bid α^i and other with maximum capacity $\bar{q}_i - q'_i$ and linear bid $\alpha^{n+i} = \beta^i$.
- Sort the slopes α^i.
- In each iteration, assign the maximum possible amount to the generator with the lowest slope until demand is met.

Finite number of iterations

Theorem

Let q^i be the vector of quantities that in iteration i assigns the maximum possible quantity to generator with the smallest slope available then q^i after at most $2|G|$ iterations is the optimal solution to the ISO problem.

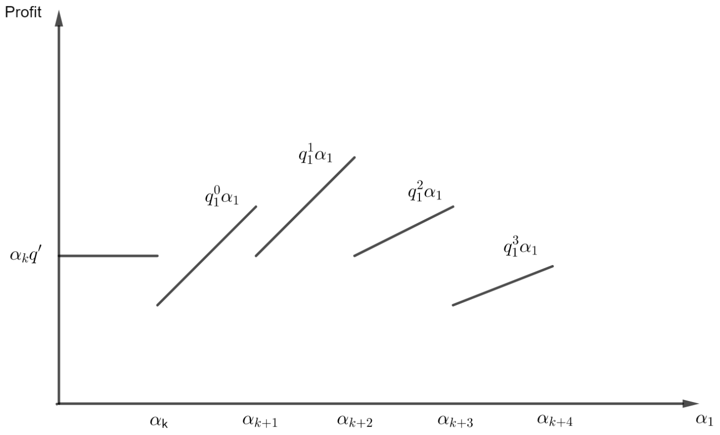
Scenarios Approach

- Fix generator i.
- Define a set of scenarios S for the remaining generators indexed by s. Which occur with probabilities $(p_s)_{s\in S}$.
- Each generator can learn from the cost functions bided by the other player and update the probabilities. Bayesian for example.
- Generator i plays the game reacting to other player's strategies. Then the bilevel problem solved by generator i is:

$$B^i(d,p) = \begin{cases} \max\limits_{(\alpha,\beta)} & \sum\limits_{s\in S} p_s \left(\lambda_s(d) q_s^i(d) - \hat{c}(q_s^i(d))\right) \\ s.t & (q_S^i, \lambda_S) \in ISO(d) \end{cases}$$

Example

- Assume that $\alpha_2 \leq \ldots \leq \alpha_{|G|}$.



Convergence result

The previous problem $B^i(d,p)$ is equivalent to his (MPEC):

$$(MPEC)^i(\alpha^{-i},d,p)=\begin{cases}\max\limits_{\alpha^i,q_S^i,\lambda_S} & \sum\limits_{s\in S}\left(p_s\lambda_s q_s^i-\hat{c}(q_s^i(d))\right)\\ s.t & \sum\limits_{n\in G} q_s^n=d,\quad s\in S\\ & 0\le q_s^n\le \bar{q},\quad n\in G, s\in S\\ & \lambda_s-\pi_s^{q^n}-\alpha_s^n\le 0,\quad n\in G, s\in S\\ & -\pi_s^{q^n}\le 0,\quad n\in G, s\in S\\ & \sum\limits_{s\in S}\left(\sum\limits_{n\in G}\alpha_s^n q_s^n-d\lambda_s+\sum\limits_{n\in G}\bar{q}\pi_s^{q^n}\right)=0\end{cases}$$

Convergence result

Consider the following problem $(MPEC)^i_{pen}(\alpha^{-i}, d, p)$ obtained when we penalize the non-linear complementarity constraint:

$$\begin{cases} \max\limits_{\alpha^i, q^i_S, \lambda_S} & \sum\limits_{s\in S}\left(p_s\left(\lambda_s q^i_s - \hat{c}(q^i_s(d))\right) - \mu\left(\sum\limits_{n\in G}\alpha^n_s q^n_s + \bar{q}\pi^{q^n}_s - d\lambda_s\right)\right) \\ s.t & \sum\limits_{n\in G} q^n_s = d, \quad s\in S \\ & 0 \leq q^n_s \leq \bar{q}, \quad n\in G, s\in S \\ & \lambda_s - \pi^{q^n}_s - \alpha^n_s \leq 0, \quad n\in G, s\in S \\ & -\pi^{q^n}_s \leq 0, \quad n\in G, s\in S \end{cases}$$

Where $\mu > 0$ is the penalty parameter.

Convergence result

Theorem: Convex Piecewise linear bids convergence

There is a penalty parameter $\bar{\mu} > 0$ such that problems $B^i(\alpha^{-i}, d, p)$ and $(MPEC)^i_{pen}$ are equivalent, and the complementary constraint holds, for every $\mu > \bar{\mu}$.

Penalty Algorithm: Solving for producer 1

Result: Expected Payoff and best strategy for generator 1
initialization;
Input: Number of players $|G|$, Maximum capacity value $\bar{q}_k$ for each player $k \in G$, the probability vector p_S of each scenario and some initial point $\tilde{q}_n^s, n \in G, s \in S$;
while *Complementary condition* $\neq 0$ **do**

  Solve penalized problem with $q_s^i = \tilde{q}_s^i$ and obtain a solution $\tilde{\alpha}, \tilde{\lambda}_s, \pi_{q_n}^s \ n \in G, s \in S$;

  Solve the ISO problem for each scenario $s \in S$, considering $\alpha = \tilde{\alpha}$ and obtain a solution $\tilde{q}_n^s$, $n \in G$;

  Increase μ

end
Return: Expected Payoff of generator 1 and best strategy.

Algorithm 1: Scenarios Approach Penalty algorithm

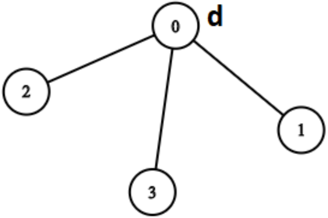
Local Search Algorithm

Result: Expected Payoff and best strategy for generator 1
Input: Number of players $|G|$, Maximum capacity value $\bar{q}_k$ for each player $k \in G$ and probability vector p_S of each scenario;
Step 1: For $k \in \{2, \dots, |G|\}$. Define the sets $I_k := \{j \in \{1, \dots, |S_k|\} : p_k(j) > 0\}$;
Step 2: Define a scenario as $s \in S = \{(t_{j_2}, \dots, t_{j_{|G|}} \in S_2 \times \dots \times S_{|G|} : j_2 \in I_2, \dots, j_{|G|} \in I_{|G|}\}$;
for $i \in |S_1|$ **do**

1. Solve the ISO's problem using our algorithm from chapter 1 ;
2. Compute the value $p_s \lambda(t_i, s) q^i_{t_i,s}$, where $s \in S$
3. Save the value $\displaystyle\sum_{s \in S} p_s \lambda(t_i, s) q^i_{t_i,s}$ as the new maximum if it is greater than the previous maximum

end
Step 3: Return Expected Payoff of generator 1 and best strategy.

Numerical Results



- Consider the 3 player asymmetric game with demand $d = 2$.

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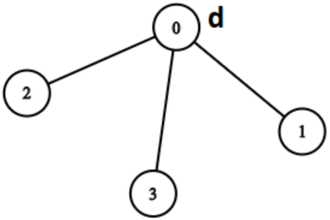

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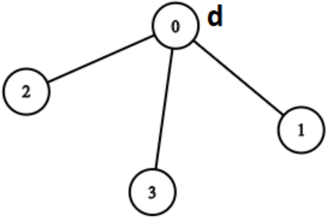

- Consider the 3 player asymmetric game with demand $d = 2$.
- Each player chooses their slopes such that : $\alpha \in [0, 1]$ and $\beta \in (\alpha, 1]$.
- True cost function slopes for each player are $(0.3, 0.5)$, $(0.4, 0.5)$ and $(0.35, 0.55)$.

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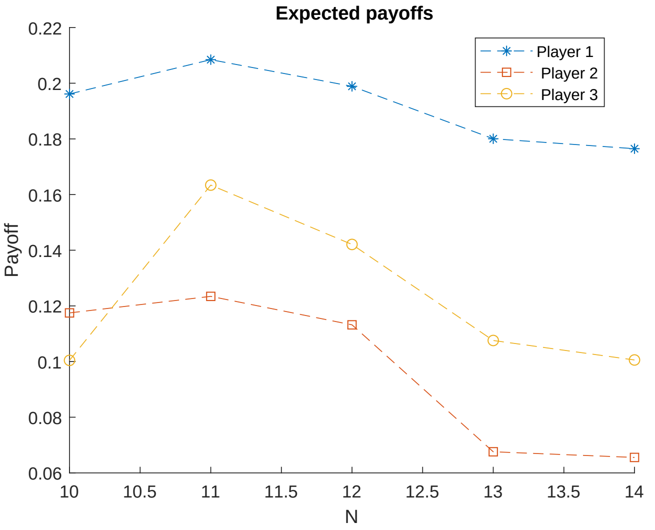

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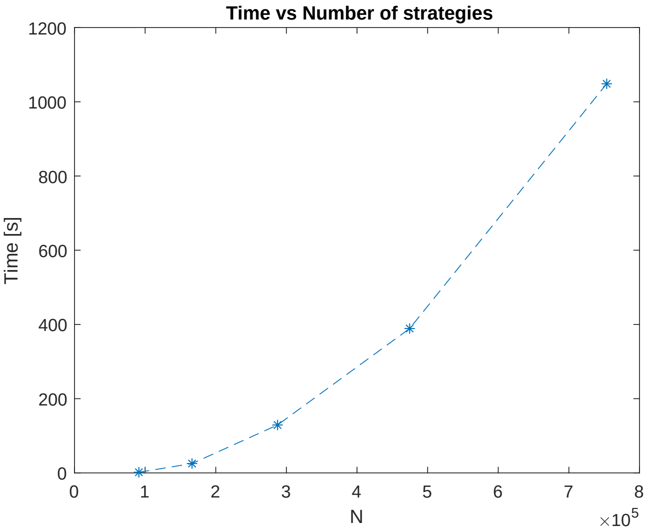
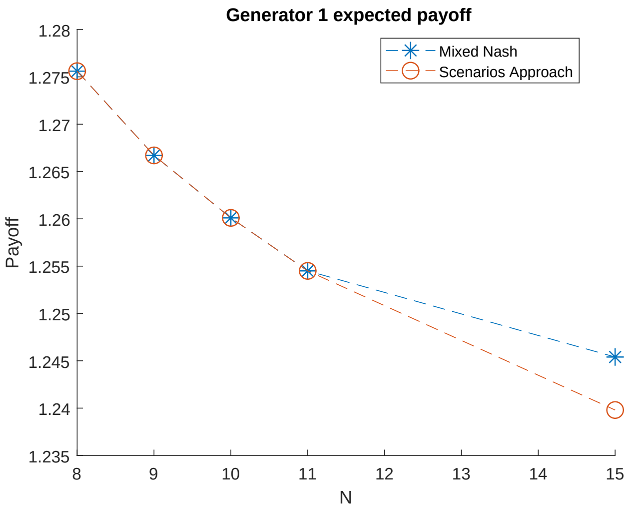


Game Theory Approach

- In this approach we find Nash equilibrium in the game using classical algorithms. Lemke, C.E., Howson (1964), B. von Stengel (2009), Herings, P.J.J., Peeters, R (2010)
- The existence of such equilibria in guaranteed in this problem (J. Escobar, A. Jofré (2007, 2012))

Numerical Results: Piecewise Linear



What about quadratic bids ?

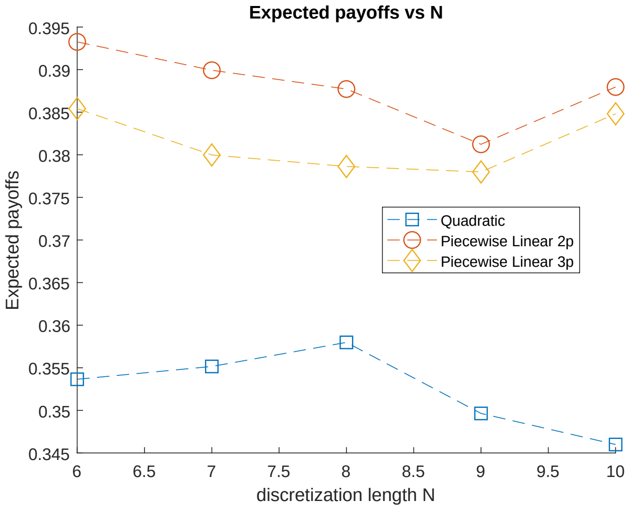
- Assume that generators choose two parameters α_n and β_n which define a quadratic function

$$c_n(q_n) = \alpha_n q_n + \beta_n q_n^2$$

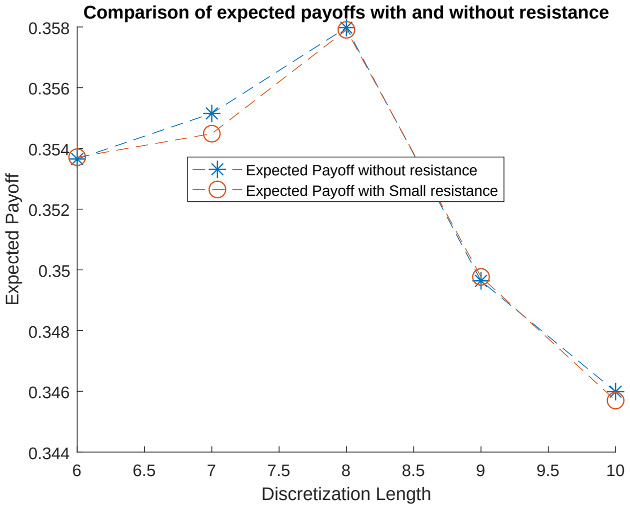
- Then for fixed d the dispatch problem is:

$$ISO(c,d) = \begin{cases} \text{mín} & \sum_{n\in G} \alpha_n q_n + \beta_n q_n^2 \\ s.t & \sum_{e\in K_n} \frac{r_e}{2} f_e^2 + d_n \leq q_n + \sum_{e\in K_n} f_e sgn(e,n), \quad n \in G \\ & \sum_{e\in K_n} \frac{r_e}{2} f_e^2 + d_n \leq \sum_{e\in K_n} f_e sgn(e,n), \quad n \notin G \\ & q_n \in [0, \bar{q}_n] \\ & f \in F \end{cases}$$

Numerical Results: Piecewise Linear and Quadratic



Numerical Results: Small resistance and no resistance



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- So, scenario approach, in which a scenario s is a set of bids and capacity values.
- Evolutionary game theory with updated information for each auction round.
- New ways to compute Nash equilibrium points !

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